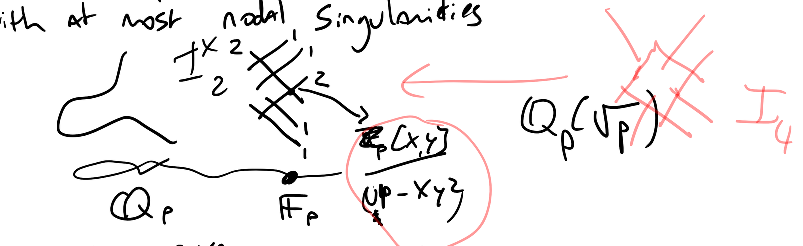
Analytic view towards semistable reduction

k NA discretely valued
k° ring of ints
$\tilde{k}$ residue char p
$\hookrightarrow \widehat{k^a}$

C/k <u>nice</u> curve $g \geq 1$
(smooth geom. conn. proj.)
model = proper relative curve$/k^\circ$
generic fiber C

good red: smooth model exists

Semistable red: $\exists$ regular model
w/ reduced special fiber
with at most nodal singularities



<u>SS red thm</u>: every curve is pot. semistable

Note that a SS model has normal crossing

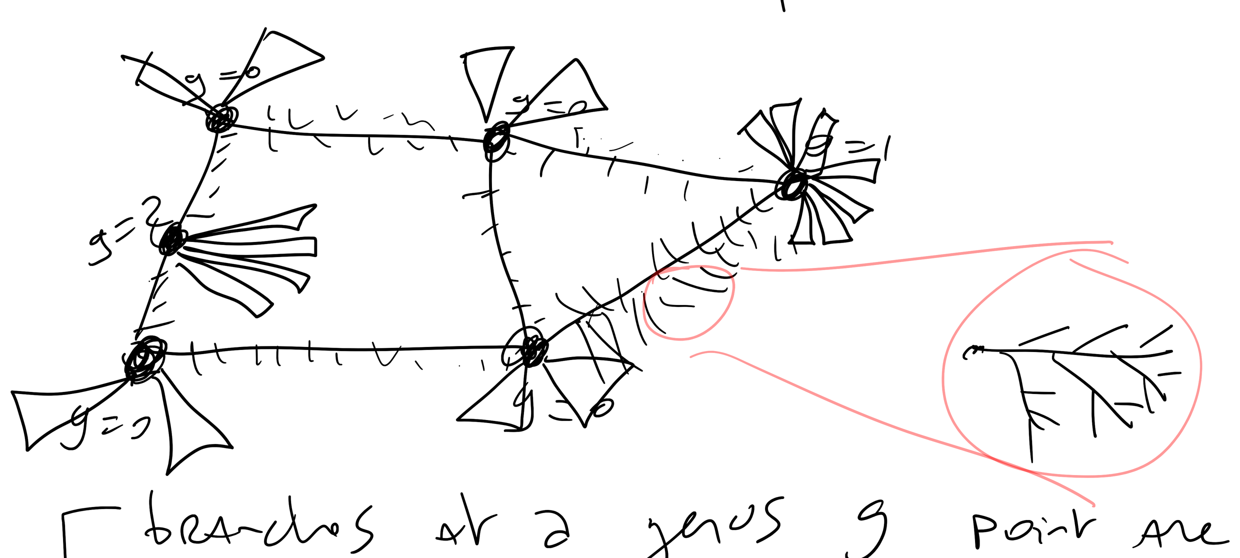
<u>Prop</u> if C has SS reduction, then the minimal nc model is semistable

<u>remarks</u>
- char 0: computation
- D & M: Analogous statement for ab. vars
- AW: study $\mathrm{Pic}(\mathcal{C}_s)$
- Saito: ℓ-adic proof

- Non-Archimedean:
  - BL → Ducros
  - Vanderput, Temkin, Arzdorf–Wewers

<u>Step 1</u> Reduce to an analogous statement for k alg closed

<u>Step 2</u> C^{an} infinite real tree



[branches at a genus g point are parametrised by the closed pts of a genus g curve over $\tilde{k}$]
$\mathcal{H}(x)/\tilde{k}$
- type 3: 2 branches
- type 1, 4: 1 branch
- type 2

<u>Step 3</u> local basis of nbhds for each pt.
<u>example</u>: every type 3 pt admits a basis of annuli

<u>Step 4</u> local ⇒ global
existence <u>skeleton</u> ⇕ <u>triangulation</u>

<u>Step 5</u>
$C^{an} \setminus$ triangulation $= \bigsqcup$ open discs $\bigsqcup$ open annuli

"$\mathrm{Spf}\left(k^\circ\{S,T\}/(ST-a)\right)$" finite for each annulus

glueing ⇝ semistable formal model

<u>Step 6</u> genuine semistable model

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k Alg closed NA field nontrivially valued

<u>Analytic Spectrum</u>

set of characters (respecting the norm)

$A \longrightarrow L$
Banach k-alg   complete NA field

$x \in \mathcal{M}(A)$ $\chi_x : A \to \mathcal{H}(x)$

$A \to L' \to L''$, $A \to L \to L''$

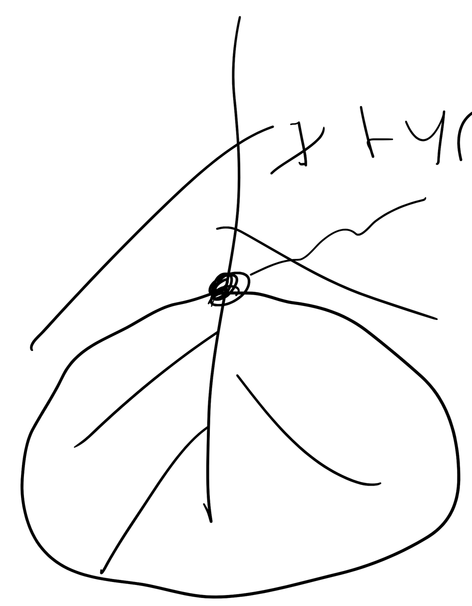
(strictly) <u>Affinoid</u>

$k\langle r_i^{-1} T\rangle = k\langle r_1^{-1}T_1, \ldots, r_n^{-1}T_n\rangle$

$r_i \in |k^\times| \rightsquigarrow$ strict Tate Alg.

$k\langle r^{-1}T\rangle \twoheadrightarrow A$
for some $r_i \in \mathbb{R}_{>0}$

<u>ex</u> nonstrictly ~~Aff.~~ Alg
$$K_r := k\langle r^{-1}T, rT^{-1}\rangle \left(= \frac{k\langle r^{-1}T, rS\rangle}{(ST-1)} \right)$$



$r \notin |k^\times|$ {
$\mathcal{H}(x) = K_r$

$D(0,r)$ $\left| \sum_{i \in \mathbb{Z}} a_i T^i \right| = \max |a_i| r^i$